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Some Recent Developments in Cosmology: Dark Matter, Inflation, and COBE

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Abstract. I review several recent developments in cosmology, focusing on the evidence for dark matter on large scales, inflationary models for the early universe, and constraints on models for large-scale structure formation from the recent COBE detection of microwave background anisotropy.

1. Introduction

In the late 1970's and early 1980's, theoretical cosmology underwent a renaissance: extrapolating concepts from particle physics, in particular the standard electroweak gauge theory, to very high energies, a framework emerged in which one could meaningfully speculate about the evolution of the very early universe. This marriage of particle physics and cosmology led to a number of remarkable developments, including models for the generation of the baryon asymmetry, the inflationary scenario, the notion that topological defects could be created in cosmological phase transitions, and predictions for non-baryonic particle dark matter, to name just a few.

In recent years, observational cosmology has been undergoing its own rebirth. There has been an explosion of information on the large-scale clustering of galaxies from redshift, peculiar velocity, and photometric surveys gathered by ground-based telescopes. Studies of rich clusters of galaxies via their gravitational lensing effects as well as X-ray emission from hot intracluster gas have started to provide new clues to the distribution of dark matter. In addition, the recent detection of large-angle anisotropies in the cosmic microwave background radiation by the COBE satellite provides the first probe of structure on very large scales, while a series of anisotropy experiments on smaller scales now have tantalizing results. On the scale of the universe itself, there has been steady progress in attempts to measure the cosmological parameters (in particular, the age, expansion rate, and mean density) as well as the light element abundances more precisely.

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As a consequence of these observational advances, cosmology is becoming data-driven in an unprecedented way: theorists no longer have the luxury of untethered speculation, but must now confront their models with an impressive array of observations. There is still debate about the reliability and interpretation of much of the data, but we are definitely entering the 'scientific' age of cosmology in the Popperian sense: those theories which are sufficiently worked out are becoming increasingly falsifiable and many will stand or fall in the coming years. This is a very healthy development for the field, but it also means we will have to become more sophisticated in confronting model predictions with data—as in particle physics, cosmologists must become, in part, phenomenologists. It is safe to say that at present the big bang framework for the large-scale evolution of the universe remains healthy, but that we still lack a standard model for the origin and evolution of structure within this framework.

In this brief review, I will focus on only several of the many topics of recent interest in cosmology: dark matter, inflation, and the implications of the recent COBE results for models of structure formation.

2. The Standard Cosmology

The standard hot Big Bang model, based on the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) spacetimes, is a remarkably successful operating hypothesis describing the evolution of the Universe on the largest scales (for a recent introduction, see Peebles 1993). It provides a framework for such observations as the Hubble law of recession of galaxies, interpreted in terms of the expansion of the universe; the abundances of the light elements, in excellent agreement with the predictions of primordial nucleosynthesis (see the contribution by Smith in this volume); and the thermal spectrum of the cosmic microwave background radiation (CMBR), as expected from a hot, dense early phase of expansion.

While homogeneity and isotropy are, strictly speaking, assumptions of the model, they rest on a strong and growing foundation of indirect observational support. The evidence for angular isotropy on large scales comes from the smallness of the CMBR quadrupole anisotropy detected by COBE, $(\Delta T/T)_{l=2} \simeq 5 \times 10^{-6}$, from the isotropy of radiation backgrounds at other wavelengths, as well as from the isotropy of deep galaxy and radio source counts. For example, the APM survey (Maddox, etal. 1990) measured the angular positions of 2 million galaxies over a solid angle of 1.3 sr in the southern sky out to an effective depth of roughly 600 h^{-1} Mpc. Here the parameter h quantifies the uncertainty about the present expansion rate: the Hubble parameter is $H_0 = 100h$ km/sec/Mpc, and observations indicate $0.4 < h < 1$ (see below). The joint probability of finding two galaxies in elements of solid angle $d\Omega_1$ and $d\Omega_2$ separated by an angle θ is given by

$$dP = N^2 d\Omega_1 d\Omega_2 [1 + w_{gg}(\theta)] \ , \tag{1}$$

where N is the mean surface density of galaxies in the survey and $w_{gg}(\theta)$, the galaxy two-point angular correlation function, measures the excess probability over random of finding a galaxy pair with this separation. If galaxies are distributed isotropically on large scales, we should find $w_{gg}(\theta) \rightarrow 0$ at large angles. In the APM survey, $w_{gg}(\theta) \sim \theta^{-0.668}$ for $\theta \lesssim 1^\circ$, but breaks below this power law at $\theta \sim 3^\circ$; for $\theta \gtrsim 6^\circ$, $|w_{gg}(\theta)| \lesssim 5 \times 10^{-4}$ and becomes lost in the noise.

Evidence for large-scale homogeneity comes in part from redshift surveys: the rms fluctuations in the spatial number density of galaxies become small when averaged over large enough scales. For example, in surveys of several thousand galaxies selected from sources in the IRAS catalog (the 1.2 Jy survey of Fisher, etal. 1993 and the QDOT survey of Efstathiou, etal. 1990a), the rms fluctuation in galaxy counts in cubical volumes of side $L = 60h^{-1}$ Mpc is of order $\delta N_{gal}/N_{gal} \simeq 0.2$ and decreases with increasing cell volume. This approach to homogeneity is consistent with that seen in the much deeper (but two-dimensional) APM survey. (In fact, the approach to homogeneity as a function of increasing lengthscale is observed to be somewhat more gradual than was expected in the popular cold dark matter model for galaxy formation–this is the famous problem of extra large-scale power, to which we'll return below.) Larger structures such as superclusters, great attractors, great voids, and long filaments do exist, and have received considerable attention from cosmologists and the press. Of particular note in this regard was the discovery of the Great Wall, extending roughly $170 \times 60 \times 5h^{-3}$ Mpc^3, by Geller and Huchra (1989) in the Center for Astrophysics (CfA) survey. But in a statistical sense, the net fluctuations in galaxy number do become small on the largest scales where they have been reliably counted in large-area redshift surveys.

This trend is qualitatively confirmed by the visual appearance of large-scale structure in the ongoing redshift survey of Shectman, etal. (1992): going considerably deeper than the CfA survey, they do not find evidence for coherent structures larger than the Great Wall. The Shectman, etal. survey plans to obtain 10,000 redshifts for galaxies in the southern sky, with a median distance of about $300h^{-1}$ Mpc, using a multifiber spectrograph with 60 fibers to simultaneously measure many redshifts in the same field; the strategy is to map out galaxies in 1.5×3 degree "bricks" which will eventually cover 1/4 of a 60×60 degree area. Advances in multi-fiber spectroscopy will be further exploited by the Sloan Digital Sky Survey, a project of Fermilab, the University of Chicago, Princeton University, the Institute for Advanced Study, Johns Hopkins University, and several Japanese universities, which aims to measure one million galaxy redshifts out to a comparable depth, covering a contiguous area of π sr in the northern sky. This survey will use a 600-fiber spectrograph to accumulate redshifts at an unprecedented rate. A concurrent photometric survey will measure angular positions for roughly 50 million galaxies.

The CMBR and galaxy observations mentioned above lend credence to the homogeneous and isotropic FRW models as a first approximation, because they indicate that the gravitational potential, and thus the perturbation to the FRW spacetime metric, associated with large-scale inhomogeneities is relatively small, $\delta\phi \sim \delta g_{\mu\nu} \sim 10^{-5}$. The FRW models are characterized by a global scale factor $a(t)$, whose dynamics is determined by the matter content of the universe through Einstein's equations,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{2}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G(\rho + 3p)}{3} + \frac{\Lambda}{3} \quad . \tag{3}$$

Here ρ is the mean energy density of matter, p is its pressure, Λ is the cosmological constant, *i.e.*, the contribution from vacuum energy, and $k = 0, 1, -1$ is the sign of the spatial curvature. Models with $k \leq 0$ are spatially infinite (open), while those with $k = 1$ are spatially finite (closed). From the Einstein equations, if $\Lambda = 0$ there is a one-to-one correspondence between the spatial geometry and the fate of the universe: open models expand forever, while closed models eventually recollapse. Observations suggest that the non-vacuum energy density of the universe is currently dominated by non-relativistic (effectively pressureless) matter, while the early universe was dominated by ultrarelativistic particles, or radiation.

The principal observable cosmological parameters of the FRW models are the Hubble parameter, $H_0 = (\dot{a}/a)_0$ (where subscript 0 denotes the present epoch), the age of the Universe, t_0, the present mass density relative to the 'critical' density of the spatially flat, Einstein-de Sitter ($k = \Lambda = 0$) model, $\Omega_0 = \rho_0/\rho_{crit} = 8\pi G\rho_0/3H_0^2$, the deceleration parameter, $q_0 = -(\ddot{a}a/\dot{a}^2)_0$, which measures the rate at which the gravitational attraction of the matter is slowing down the expansion, and the contribution of the cosmological constant to the present expansion rate, $\lambda_0 = \Lambda/3H_0^2$. From the Einstein equations, these parameters are related by

$$1 = \Omega_0 + \lambda_0 - \frac{k}{a_0^2 H_0^2} \tag{4}$$

$$q_0 = \frac{\Omega_0}{2} - \lambda_0 \quad . \tag{5}$$

In addition, the age of the universe is related to the other parameters through an expression of the form $H_0 t_0 = f(\Omega_0, \lambda_0)$; for a matter-dominated universe with $\Lambda = 0$, f falls monotonically with increasing Ω_0, and two useful limits are $f(0,0) = 1$ and $f(1,0) = 2/3$. More generally, over the range $0 < \Omega_0 \leq 1$, $k \leq 0$, $\Omega_0 - 3\lambda_0/7 \leq 1$, an excellent approximation is

$$H_0 t_0 \simeq \frac{2}{3} \frac{\sinh^{-1}(\sqrt{(1-\Omega_a)/\Omega_a})}{\sqrt{1-\Omega_a}} \tag{6}$$

where

$$\Omega_a = \Omega_0 - 0.3(\Omega_0 + \lambda_0) + 0.3 \quad . \tag{7}$$

(Carroll, Press, and Turner 1992).

Much early effort was spent trying to measure or constrain the parameters q_0 and λ_0 through the classical 'cosmological tests', such as the Hubble diagram, angular size as a function of redshift (for a recent discussion using radio jets see Krauss and Schramm 1993), and galaxy counts as a function of redshift and apparent brightness. For example, to construct the Hubble diagram, one measures the apparent brightness of a well-defined sample of objects (say, the brightest galaxies in clusters) as a function of the object's redshift; for galaxies of fixed intrinsic luminosity, the scaling of apparent magnitude with redshift is a function of the cosmological parameters. Unfortunately, galaxies at large distances, where the distinction between model parameters becomes observable, are seen when they were much younger than their nearby counterparts, so a model for galaxy luminosity evolution must be used to interpret the results. Significant progress has been made in understanding galaxy evolution, and there is hope that the effects of evolution and cosmology might be disentangled in coming years, but these tests currently do not place stringent constraints on the cosmological parameters (see Koo and Kron 1992). A recent twist along these lines was added by Fukugita and Turner (1991), who pointed out that the probability that a quasar at a given redshift is gravitationally lensed by a foreground galaxy is a sensitive test for the cosmological constant; based on surveys for lensed quasars, they inferred the bound $\lambda_0 \lesssim 0.9$ in the case of a spatially flat ($k = 0$)

universe (for a review on Λ, see Carroll, Press, and Turner 1992). While this limit is somewhat controversial, as the statistics of gravitational lenses grows with larger, more carefully controlled quasar surveys, this kind of test should become increasingly useful.

The Hubble parameter relates the observed recession velocity v_r or redshift z of a galaxy to its distance d: for $v_r \ll c$, the recession velocity is $v_r = cz = H_0 d + v_{pec}$, where $1 + z = (\lambda_{obs}/\lambda_{em})$ is the ratio of the observed wavelength of a spectral feature to its wavelength at emission (or at rest in the laboratory), and v_{pec} is the peculiar radial velocity of the galaxy with respect to the Hubble flow, usually assumed to arise from gravitational clustering. Galaxy redshifts can be measured quite accurately, so all the difficulty in determining H_0 resides in finding reliable distance indicators for extragalactic objects at distances large enough that the Hubble term dominates over the peculiar motion. Observed peculiar velocities are typically of order 300 km/sec, so that distance measurements beyond 40 Mpc or more (recession velocities above 4000 km/sec) are required for reasonable accuracy.

A wide variety of techniques has been used to establish an extragalactic distance scale, and this is reflected in the spread of results for H_0, roughly $40 - 100$ km/sec/Mpc (for a critical review, see Jacoby, etal. 1992). Distance estimates made using methods such as the Tully-Fisher relation between 21-cm rotation speed and infrared luminosity for spiral galaxies, calibrated by observations of Cepheid variable stars in several nearby galaxies, have yielded high values for the expansion rate, roughly $H_0 = 80 \pm 10$ km/sec/Mpc. Two newer methods, planetary nebula luminosity functions (Jacoby, etal. 1990) and galaxy surface brightness fluctuations (Tonry 1991) yield values for H_0 in this range as well, and are being further developed. On the other hand, methods using Type Ia supernovae as standard candles have yielded low values. As discussed by Wheeler in these proceedings, SNe Ia are thought to be the explosions of white dwarfs which accrete matter from binary companions until they reach the Chandrasekhar mass, and there is some evidence that they form a homogeneous class (Branch and Tammann 1992); they also have the advantage that they can be observed to great distances. A recent calibration of the absolute luminosity of SN Ia 1937C using observations of Cepheids in the nearby galaxy IC 4182 yields $H_0 = 45 \pm 9$ km/sec/Mpc (Sandage, etal. 1992). In the future, Hubble Space Telescope observations of Cepheids in other nearby galaxies which are hosts to SNe Ia should help improve the situation. The recent discovery of a probable Type Ia supernova at $z = 0.46$ (Pennypacker etal. 1992) also raises hopes that a sample of SNe Ia at $z \sim 0.5$ could significantly constrain q_0, provided the dispersion in SNe Ia luminosities is sufficiently narrow.

There are also a variety of methods being employed to measure the distances of extragalactic objects directly, bypassing the extragalactic distance ladder built up from Cepheids. Using the expanding photosphere method, Schmidt, Kirshner, and Eastman (see the contribution by Kirshner in these proceedings) have determined the distances to 10 type II supernovae at distances up to 120 Mpc, and obtain $H_0 = 60 \pm 10$. Other 'direct' methods which hold promise include measurement of the Sunyaev-Zel'dovich effect, due to the Compton upscattering of CMBR photons by hot gas in rich clusters (Birkinshaw 1991), and the differential time delay between images in gravitationally lensed quasars (Blandford and Narayan 1992).

Three methods have traditionally been used to infer the age of the Universe, t_0. Nuclear cosmochronology, based on radioactive dating of r-process elements, generally indicates $t_0 = 10 - 20$ Gyr (Schramm 1990), while the observed paucity of very cool white dwarfs implies that the age of the galactic disk is about $t_0 \simeq 10 \pm 2$ Gyr (Winget etal 1987). By far the most extensively studied technique is the determination of the ages of the oldest globular clusters in the galaxy, with the current typical estimate $t_{gc} = (13 - 15) \pm 3$ Gyr (Renzini 1992). The largest source of error is apparently uncertainty in the distances to the globular clusters. It is hoped that observations with the corrected Hubble Space Telescope mirror, scheduled for a repair mission in late 1993, could reduce the uncertainty in t_{gc} to as little 10%.

3. Dark Matter and Ω_0

It is convenient to parameterize the mass density of the universe in terms of the mass-to-light ratio, say in the V_T band, $\Upsilon = (M/L)/(M_\odot/L_\odot)$. Dividing the present critical density, $\rho_c = 3H_0^2/8\pi G = 1.88h^2$ gm cm^{-3} $= 2.8 \times 10^{11} h^2 M_\odot$ Mpc^{-3} by the observed mean luminosity density $j_{V_T} \simeq 2.4 \times 10^8 h L_\odot$ Mpc^{-3}, the critical mass-to-light ratio for the $\Omega_0 = 1$ universe is $\Upsilon_c \simeq 1200h$, and the cosmic density parameter can be expressed as $\Omega_0 = 8 \times 10^{-4}\ h^{-1}\Upsilon$. The mass-to-light ratio in the solar neighborhood is approximately $\Upsilon = 5$, while the central cores of elliptical galaxies yield $\Upsilon = 12h$, so the density of luminous matter (that is, of matter associated with typical stellar populations) is inferred to be $\Omega_{lum} \sim 0.007$. However, it is well known that the luminous parts of galaxies are not the whole story: there is strong evidence from flat spiral galaxy rotation curves and from a variety of observations of galaxy clusters that there is a substantial amount of dark matter associated with galaxies and clusters.

On the scale of individual galaxies, the observation of high proper motion stars in the solar neighborhood (presumably bound to the Galaxy) implies that the local value of the galactic escape velocity exceeds 450–500 km/sec. In a truncated isothermal sphere model of the galaxy halo, this implies that the total mass-to-light ratio for the Milky Way is at least $\Upsilon_{MW} \gtrsim 30$. This is consistent with the requirement that distant globular clusters and satellite galaxies are bound to the Galaxy, as well as with mass-to-light ratios inferred from flat rotation curves in other spiral galaxies (for a recent review, see Fich and Tremaine 1991). If these systems are typical of the universe, we infer $\Omega_0 \gtrsim 0.02h^{-1}$.

It is interesting to compare these values with the baryon density Ω_B inferred from primordial nucleosynthesis (see the contribution by Smith, this volume), which recently has been restricted to the range $0.010 < \Omega_B h^2 < 0.015$ (Walker etal. 1991, Smith etal. 1993). Comparison with Ω_{lum} above suggests that some or most of the baryons are dark or in underluminous populations. Furthermore, comparison with the escape speed and rotation curve bounds shows that baryons could constitute some or all of the dark matter in galaxy halos. One possibility would be degenerate brown dwarfs, substellar ($M < 0.08M_\odot$) objects which did not reach sufficiently high temperature to burn hydrogen. Currently three independent groups are searching for halo dwarfs (which have been dubbed MACHOS, for massive compact halo objects); the signature is a microlensing event, in which a background star, say in the LMC or the bulge of the Milky Way, characteristically brightens and fades as a MACHO passes near its line of sight. Although they are distinguishable from ordinary variable and flare stars, such events would be intrinsically rare, so a large number of stars must be accurately monitored. The MACHO project, a collaboration between Lawrence Livermore National Laboratory, the

UC Berkeley Center for Particle Astrophysics, and Mt. Stromlo Observatory, currently obtains CCD photometry for 2 million stars per night, and has discovered a number of periodic variable stars. French and Polish groups are also making progress. In the next few years, these efforts should either discover or place important constraints on baryonic dark matter (see Griest 1992 for a recent overview).

Moving to larger scales, for clusters of galaxies the traditional dynamical method of estimating cluster masses and mass–to–light ratios, first used by Zwicky who discovered the 'missing mass' problem in the 1930's, has been to apply the virial theorem to measured cluster velocity dispersions, $M_{tot} \propto \langle v^2 \rangle / \langle R_{ij}^2 \rangle$, where $\langle v^2 \rangle$ is the velocity dispersion of the galaxies in a cluster and R_{ij} is the separation between them. This method assumes that galaxies trace the cluster mass and that the galaxy velocity distribution is isotropic, both of which may be poor approximations (see the contributions in Oegerle etal. 1990). Independent information on the dark matter distribution in the inner cores of clusters comes from the giant luminous arcs and arclets, high redshift galaxies gravitationally lensed by foreground clusters (Lynds and Petrosian 1986, Soucail, etal. 1987). These arcs are formed when a galaxy is nearly imaged into an Einstein ring. Measurements of the cluster and background galaxy redshifts yield an estimate of the cluster mass within the impact parameter of the lens; for most of the cases studied, these estimates are in reasonable agreement with the mass–to–light ratios inferred from the virial theorem, but it should be noted that the lens observations probe only the inner few hundred kpc of the clusters. Typical inferred cluster values are $\Upsilon \sim 100 - 250h$, which would imply $\Omega_0 \sim 0.1 - 0.2$. Since clusters are rare objects, occupying a very small fraction of the universe, one expects this to be a lower bound, in which case some form of *non-baryonic* dark matter is probably required, given the limits on Ω_B.

X-ray observations, most recently by the ROSAT satellite (e.g., White etal. 1993), have begun to map the density and temperature profiles of the hot gas which permeates many clusters. Since the gas is in hydrostatic equilibrium, it can be used to trace the cluster mass distribution (including dark matter) directly,

$$M_{tot}(r) = -\frac{kT(r)}{G\mu m_p}\left(\frac{d\ln n}{d\ln r} + \frac{d\ln T}{d\ln r}\right) \tag{8}$$

where μ is the mean molecular weight of the gas, and $n(r)$ and $T(r)$ are the gas density and temperature. Cluster masses inferred from X-ray observations are generally comparable to but ~30% less than virial estimates. Consistent with this, the X-ray measurements indicate that clusters are surprisingly baryon-rich, in the sense that the gas constitutes typically $10h^{-3/2}\%$ of the inferred binding mass within approximately $1h^{-1}$ Mpc of the center of a rich cluster like Coma. Moreover, the dark matter tends to be *more* centrally concentrated than the gas out to this scale, suggesting that the gas fraction of the whole cluster is at least as large as the value above (Sarazin 1986, Hughes 1989). If this ratio is representative of the baryon mass fraction of the universe, then the nucleosynthesis bound on Ω_B would imply the *upper* limit $\Omega_0 < 0.15h^{-1/2}$ (Cf. White and Frenk 1991). (Including the baryon component in cluster galaxies would slightly strengthen this limit.) This has been taken as evidence against the universe having closure density ($\Omega_0 = 1$) and would require advocates of inflation (which implies $k = 0$) to fall back on a cosmological constant (see Eqn.(4)). The other possibility would be loosen the nucleosynthesis bounds on Ω_B through some non-standard scenario such as inhomogeneous nucleosynthesis, but the upper bound on Ω_B is not raised sufficiently model to get around the argument above (Walker etal. 1991 and references therein). On the other hand, several caveats about the baryon fraction inferred from clusters should be kept in mind. While numerical simulations of cluster formation with baryons and cold dark matter do indicate that a spatially resolved X-ray profile which extends sufficiently far in radius does give an accurate estimate of the total cluster mass within that radius (Evrard 1993), they also suggest that $M_{tot}(r)$ can be substantially underestimated (and thus the baryon fraction overestimated) at smaller radii (Tsai 1992, Babul and Katz 1993). A second cause for suspicion is the significant dispersion in the inferred gas fraction of different clusters. In particular, Mushotzky (1992) has noted that clusters with higher X-ray temperatures (and therefore larger total masses) appear to have a larger fraction of their total mass in gas. This trend, coupled with the steeply falling distribution function of cluster temperatures, $dn_c/dT \sim T^{-5}$ for $3 < kT < 10$ keV (Edge etal. 1990), suggests that the mean gas (and baryon) fraction may be substantially below the value for Coma, since the mean is dominated by the more numerous cooler clusters ($kT_{Coma} \simeq 7$ keV). This would raise the derived upper bound on Ω_0 closer to unity and suggests that massive clusters like Coma may not be representative of the baryon fraction of the Universe.

Moving to still larger scales, the deviations from the Hubble flow have been used to infer the cosmic density over scales of order $50h^{-1}$ Mpc. The basic idea is to compare samples of the density perturbation field and the peculiar velocity field covering the same volume; assuming they arise gravitationally, the proportionality between them depends on the rate of growth of the density fluctuations, which in turn depends on Ω. On large scales, as discussed above, the rms density fluctuations are small, so linear perturbation theory away from the FRW spacetime is an accurate first approximation. In this case one finds the relation

$$\nabla \cdot \mathbf{v}_{pec} = -H_0 \Omega_0^{0.6} \delta \quad , \tag{9}$$

where the density field $\delta(\mathbf{x}) = (\rho(\mathbf{x}) - \bar{\rho})/\bar{\rho}$, and the perturbation growth rate enters through $d\ln\delta/d\ln a \simeq \Omega^{0.6}$. If one expresses distances d in terms of their equivalent Hubble velocities, $v = H_0 d$, then H_0 drops out of Eqn.(9), so the uncertainty in the Hubble parameter does not undermine this method. A number of different approaches have been used to extract Ω_0 in this way, and they have all given consistently high answers. An apparent obstacle is that observations provide only the radial component of peculiar velocities. In response, Bertschinger and Dekel (1989) developed the POTENT method, using the irrotational nature of the induced velocities to reconstruct the three-dimensional velocity field from sparse, noisy samples of radial peculiar velocities obtained by the Tully-Fisher and related methods. Dekel, etal. (1992) have compared the divergence of the reconstructed velocity field with the density field inferred from the 1.9 Jy redshift survey of IRAS galaxies. The qualitative topographical appearance of the smoothed POTENT and IRAS density fields is strikingly similar–the same large-scale peaks, ridges, and voids are seen in both. This correlation suggests that galaxies do broadly trace the mass distribution on large scales, in the sense that more galaxies are found in regions of high mass density, but the galaxy distribution may be 'biased' with respect to the mass. In the simplest linear bias model, the smoothed galaxy and mass density fields are assumed to be proportional, $\delta_{gal}(\mathbf{x}) = b_{gal}\delta(\mathbf{x})$, where b_{gal} is the bias factor, taken to be constant for a given class of galaxies. Since δ_{gal} is what is observed, the POTENT-IRAS comparison actually constrains the combination $\Omega_0^{0.6}/b_{IRAS}$. Dekel, etal. (1992) obtained $\Omega_0^{0.6}/b_{IRAS} = 1.28^{+0.75}_{-0.59}$ at 95% confidence. For a bias factor of order unity, this is consistent with $\Omega_0 = 1$, which is pleasing to theorists and also buttresses the case for non-baryonic dark matter. It is well to keep in mind, however, that biasing is presumably a complex process associated with the non-linear stages of galaxy formation, so that the proportionality of the galaxy and density fields may be non-linear and/or scale-dependent.

It is convenient to distinguish two broad classes of non-baryonic dark matter, hot and cold, on the basis of their clustering properties. The prototypical hot dark matter candidate is a light neutrino with mass $m_\nu \simeq 20$ eV. Since they are relativistic ($m_\nu \lesssim T$) until relatively recent epochs, light neutrinos would free-stream out of and damp out density perturbations up to the scale of galaxy clusters. Galaxies would form after clusters via fragmentation ('top down'). Due to phase-space constraints (Tremaine and Gunn 1979), light neutrinos would not cluster significantly on the scale of galaxies: baryons would constitute the predominant dark matter in galaxy halos, while neutrinos would dominate in clusters. Cold dark matter, on the other hand, is defined to have negligible free-streaming length–it clusters on all scales. In cold dark matter models, structure generally forms hierarchically, with smaller clumps merging to form larger ones ('bottom up'). For this reason, theorists since the early '80's have tended to prefer cold over hot dark matter, but it is worth noting the recently surging popularity of a mix 'n match scenario: a combination of 70% cold and 30% hot dark matter (say, with $m_\nu \simeq 7$ eV) produces a favorable spectrum of large-scale density perturbations in the context of inflation, with some apparent advantages over pure cold dark matter (for a recent overview with references to the burgeoning literature, see Pogosyan and Starobinsky 1993).

The theoretically favorite candidates for cold dark matter are weakly interacting massive particles (WIMPs), with masses generally in the range 20 – 150 GeV, and the axion, an ultra-light pseudoscalar with a mass of order 10^{-5} eV. The most attractive WIMP candidate is the neutralino, a supersymmetric fermionic partner of the standard model bosons; its weak annihilation rate in the early universe naturally leaves it with an abundance comparable to the present critical density (for a recent review see Roszkowski 1993). The axion is the pseudo-Nambu-Goldstone boson associated with spontaneous breakdown of a global $U(1)$ symmetry (the Peccei-Quinn symmetry) introduced to explain why the strong interactions conserve CP. The global symmetry is spontaneously broken at some large mass scale f_{PQ}, through the vacuum expectation value of a complex scalar field, $\langle\Phi\rangle = f_{PQ}\exp(ia/f_{PQ})/\sqrt{2}$. At energies below the scale f_{PQ}, the only relevant degree of freedom is the massless axion field a, the angular mode around the bottom of the Φ potential. At a much lower energy scale, $\Lambda_{QCD} \sim 100$ MeV, the symmetry is *explicitly* broken when QCD becomes strong, and the axion obtains a periodic potential of height $\sim \Lambda^4_{QCD}$. In 'invisible' axion models with Peccei-Quinn scale $f_{PQ} \sim 10^{12}$ GeV, the resulting axion mass is $m_a \sim \Lambda^2_{QCD}/f_{PQ} \sim 10^{-5}$ eV and $\Omega_a \sim 1$. Although light, invisible axions interact so weakly, with cross-section $\sigma \sim 1/f^2_{PQ}$, that they were never in thermal equilibrium: they form as a cold Bose condensate (for axion reviews, see Kim 1987, Turner 1990, Raffelt 1990).

Accelerator searches for neutrino mixing and beta-decay experiments on neutrino mass should provide useful constraints on the possibility of neutrino dark matter. Active experimental efforts are also underway to detect both WIMPs and axions. Direct WIMP detection looks for the signals produced when a halo WIMP collides with a nucleus in a kg-size cryogenic crystal, depositing of order 10 keV in ionization and phonons (detector schemes based on scintillation and excitations of superfluids and superconductors are also being developed). Indirect WIMP detectors search for high energy neutrinos produced when WIMPs annihilate in the Sun and the Earth; large underground or underwater detectors currently in place or under development with sensitivity to WIMP annihilations include Kamiokande, MACRO, AMANDA, and DUMAND (see Griest 1992). Accelerator searches for supersymmetry also will constrain the neutralino parameter space. A large-scale axion search based at Livermore, expected to come on-line in 1994, will search for resonant conversion of halo axions to microwave photons in a cavity with a strong magnetic field. A scaled-up version of an idea originally proposed by Sikivie, this detector should reach the cosmologically interesting region of axion couplings for the first time (Van Bibber 1992).

Finally, given the paucity of direct evidence for dark matter, it is probably healthy to keep an open mind to alternatives. While it is natural to ascribe flat galaxy rotation curves and large cluster velocity dispersions to unseen matter, Milgrom (1988 and references therein) and others have argued that they may instead signal a breakdown of Newton's law of inertia at very low acceleration. The extent to which Milgrom's modified Newtonian dynamics (MOND) accounts for all the phenomena normally imputed to dark matter is controversial (Lake 1989, Milgrom 1991, Gerbal etal. 1992), and a full theory with which one could explore cosmology has been lacking. Nevertheless, at a minimum it provides a useful challenge to the accepted dogma. On a relatively more conservative side, the possibility that the dark matter interacts by other long-range forces in addition to gravity has recently been explored (Frieman and Gradwohl 1991). Such interactions are significantly constrained by galaxy and cluster observations, but could nevertheless have interesting implications for structure formation and biasing (Gradwohl and Frieman 1992).

With this brief survey in hand, it is useful to pause and place these numbers for the cosmological parameters in theoretical context. If an extended period of inflation took place in the early universe (see below), then the spatial geometry should now be observationally indistinguishable from $k = 0$. If the cosmological constant vanishes, from Eqn.(4) spatial flatness implies $\Omega_0 = 1$ (with the concomitant requirement of non-baryonic dark matter), and thus $t_0 = 2/3H_0 = 6.5h^{-1} \times 10^9$ yr. This is uncomfortably low compared to globular cluster ages unless $h \lesssim 0.5$ ($t_0 \gtrsim 13$ Gyr) and definitely problematic unless $h < 0.65$ ($t_0 > 10$ Gyr), still on the low side of the H_0 observations. However, a non-vanishing λ_0 is certainly allowed at some level by observations and has sporadically come into vogue, most recently to alleviate both this age problem and the large-scale power problem for inflationary cosmology (Efstathiou etal 1990b, Turner 1991). For example, in a flat model with $\Omega_0 = 0.25$, $\lambda_0 = 0.75$, Eqn.(6) gives $t_0 \simeq 1/H_0 = 9.75h^{-1}$ Gyr, consistent with the lower bound $t_{gc} > 10$ Gyr for the entire observed range of H_0, and yielding a healthy, $t_0 > 13$ Gyr-old universe for $h < 0.75$. This lower value of Ω_0 is consistent with the dynamical estimates from clusters, but somewhat below the recent estimates from large-scale flows. Since there is currently little theoretical guidance as to why $|\lambda_0|$ is as small as it is, and no firm proof that it should vanish, it is probably best to keep an open mind, although the fact that we would be living just at the epoch when Ω_0 is comparable to λ_0 might seem to beg for explanation. The third possibility is that theoretical prejudice is wrong, and that we live in an open, low-density, perhaps purely baryonic universe with negligible Λ, in which case the globular cluster age range is also compatible with somewhat larger values of H_0. The challenge in this case is to explain the large-scale flows (or attribute them to systematic distance errors) and to form large-scale structure without violating CMBR anisotropy constraints.

4. Inflation, Large-scale Structure, and COBE

The inflationary scenario is a remarkably elegant idea: if the early universe undergoes an epoch of accelerated expansion during which the Robertson-Walker scale factor $a(t)$ increases by a factor of at least e^{60}, then a small causally connected region grows to a sufficiently large size to explain several puzzles of the standard cosmology. As a bonus, quantum fluctuations of the scalar field that drives inflation (the *inflaton*) can causally generate large-scale density fluctuations, which are required for galaxy formation (for reviews see Kolb and Turner 1990, Olive 1990).

Despite its theoretical appeal, inflation faces two stiff challenges: the small coupling problem and the large-scale structure problem. The first is endemic to all inflation models: in order for the fluctuations generated during inflation to be consistent with the CMBR anisotropies observed by COBE, the inflaton must be extremely weakly self-coupled, with effective quartic self-coupling $\lambda_\phi \sim 10^{-13}$. For this reason, there is as yet no consensus on a particular brand of inflation from the standpoint of particle physics, and there are many models on the market. The challenge is to find models in which this small number arises naturally. In particular, since inflation takes place *relatively* close to the Planck scale, it would be preferable to find the inflaton in particle physics models which are "strongly natural", that is, which have no small numbers in the fundamental Lagrangian.

In a strongly natural gauge theory, all small dimensionless parameters ultimately arise dynamically. In particular, in an asymptotically free theory, the scale M_1, at which a running coupling constant $\alpha(M)$ becomes unity, is $M_1 \sim M_2 e^{-1/\alpha(M_2)}$, where M_2 is the fundamental mass scale in the theory. When coupled to a spontaneously broken global symmetry, there emerges a scalar field whose self-coupling λ_ϕ arises from this ratio of mass scales, $\lambda_\phi \sim (M_1/M_2)^q$; for example, in the model to be discussed below, $q = 4$. As a result, in such models, λ_ϕ is naturally exponentially suppressed, $\lambda_\phi \sim e^{-q/\alpha}$.

An example of this kind in particle physics, namely, a scalar field with small self-coupling, is provided by the axion. In the invisible axion model discussed above, the induced axion self-coupling is extremely small: $\lambda_a \sim (\Lambda_{QCD}/f_{PQ})^4 \sim 10^{-52}$. This small number simply reflects the hierarchy between the QCD and Peccei-Quinn scales, which arises from the slow logarithmic running of the $SU(3)_c$ strong gauge coupling constant α_{QCD}.

Pseudo-Nambu-Goldstone bosons like the axion are ubiquitous in particle physics models: they arise whenever an approximate global symmetry is spontaneously broken. The argument above suggests that they are good candidates for the role of the inflaton, since their self-coupling can be naturally small. In the simplest such 'natural inflation' models (Freese, Frieman, and Olinto 1990), the inflaton potential is of the form $V(\phi) = \Lambda^4[1 \pm \cos(\phi/f)]$. This can give rise to successful inflation if $f \gtrsim 0.3 M_{Pl} \simeq 3 \times 10^{18}$ GeV and $\Lambda \sim M_{GUT} \sim 10^{15}$ GeV; in this case, the effective quartic coupling is $\lambda_\phi \sim (\Lambda/f)^4 \sim 10^{-13}$, as required for the density fluctuation amplitude. Such mass scales arise in particle physics models with a gauge group that becomes strongly interacting at the grand unification (GUT) scale.

This brings us to the second challenge facing inflation, that of large-scale structure. In the standard lore of inflation, the adiabatic density fluctuations generated have a nearly scale-invariant (SI) spectrum (first suggested on phenomenological grounds by Harrison, Zel'dovich, and Peebles and Yu in the early 1970's). However, as we will see below, the SI spectrum with standard cold dark matter ($\Omega_0 = 1$, $h = 0.5$) appears to have too little relative power on large scales to adequately explain the observed large-scale clustering of galaxies and clusters–this is the extra-power problem mentioned in §2.

Several possible solutions to this problem have been considered. To put them into context, let us first introduce some nomenclature. It is convenient to consider the Fourier transform of the density field, $\delta_k = \int d^3x \exp(i\vec{k}\cdot\vec{x})\ \delta(\vec{x})$, in terms of which the density power spectrum is defined as $P_\rho(k) \equiv |\delta_k|^2$. For the SI spectrum, the *primordial* spectrum is linear in wavenumber, $|\delta_k(t_i)|^2 = Ak$, and the present spectrum is related to the primordial one through the transfer function, $|\delta_k(t_0)|^2 = T^2(k)|\delta_k(t_i)|^2$. The transfer function $T(k)$ encodes the scale-dependence of the gravitational evolution of the perturbation modes; it depends on the nature of the dark matter (hot or cold), its density (Ω_{DM}) and that of baryons (Ω_B), and the Hubble parameter $H_0 = 100\ h$ km/sec/Mpc. On scales which enter the Hubble radius after the universe becomes matter-dominated, $k \ll k_{eq} \simeq 0.2\Omega_0 h^2$ Mpc^{-1}, all perturbations undergo the same growth rate, so the transfer function $T^2(k) \simeq 1$; on small scales, $k \gg k_{eq}$, it bends over to $T^2(k) \sim k^{-4}$ as $k \to \infty$, reflecting the suppressed growth of fluctuations which cross inside the Hubble radius while the universe is still radiation-dominated. For standard cold dark matter (CDM), we take $h = 0.5$ and assume negligible baryon density, $\Omega_B \ll 1$, $\Omega_{cold} = 1$, leading to the characteristic scale $k_{eq} \simeq 0.05$ Mpc^{-1}. For hot dark matter, on the other hand, in the absence of seed perturbations such as cosmic strings, the transfer function is also exponentially damped by neutrino free-streaming on wavenumbers larger than $k_\nu \simeq 0.1(m_\nu/20\text{eV})$ Mpc^{-1}. Finally, the present density spectrum is related to the galaxy power spectrum by a bias prescription; for the linear bias model discussed in §3,

$$P_{gal}(k) = b_{gal}^2 P_\rho(k) = b_{gal}^2 T^2(k)|\delta_k(t_i)|^2 \quad . \tag{10}$$

Taking the standard CDM+SI model as a fiducial reference, solutions to the extra-power problem can be classified according to which element on the right hand side of Eqn.(10) one tinkers with. In all cases, the aim is to flatten the shape of the spectrum at intermediate wavenumbers $k \sim 0.05$ h Mpc^{-1} by increasing the relative power on these scales compared to smaller wavelengths. For example, one can abandon standard CDM and increase the characteristic wavelength where the transfer function $T(k)$ bends down, providing more relative large-scale power, by reducing Ω_0 from unity (keeping spatial flatness by introducing a cosmological constant) or by employing a mixture of hot and cold dark matter, as mentioned in §3. Alternatively, one can admit a more complex scheme for biasing in which the bias factor is scale-dependent, $b_g \to b_g(k)$, and increases at large scales (Babul and White 1991, Bower etal. 1993). The third possibility is to retain linear bias and standard CDM but consider *non*-scale-invariant primordial perturbation spectra which have more relative power on large scales than the SI spectrum. While this runs counter to the standard lore, in fact the perturbations generated in several models of inflation can deviate significantly from the SI spectrum. For example, in natural inflation the primordial spectrum is a non-SI power law, $P(k) \propto k^{n_s}$, with $n_s \simeq 1 - (M_{Pl}^2/8\pi f^2)$ (where the SI spectrum corresponds to $n_s = 1$). Extended and power law inflation also give rise to power law spectra with index $n_s < 1$ (Kolb etal. 1990, Vittorio etal. 1988). (On the other hand, for chaotic inflation, $n_s \gtrsim 0.95$.) As an example of a class of models with extra large-scale power, it is interesting to consider the implications of and constraints upon such non-SI power law spectra in the context of standard CDM and in light of the recent COBE measurement of the large-angle CMBR anisotropy (Adams, etal. 1993, Liddle etal. 1992, Cen etal. 1992, Gelb etal. 1993).

For these models, the density power spectra form a two-parameter family characterized by the spectral index n_s and the normalization A. Instead of A, it is common to normalize spectra by the *rms* linear mass fluctuation in spheres of radius 8 h^{-1} Mpc, $\sigma_8 \equiv \langle(\delta M/M)^2\rangle^{1/2}_{R=8h^{-1}\mathrm{Mpc}}$, where

$$\sigma_R^2 = \frac{1}{2\pi^2}\int_0^\infty dk k^2 P(k) W^2(kR) \ , \tag{11}$$

and the window function

$$W(kR) = \frac{3}{(kR)^3}(\sin kR - kR\cos kR) \tag{12}$$

filters out the contribution from small scales. Redshift surveys of optically selected galaxies (in particular the CfA survey) indicate that the variance in galaxy counts on this scale is of order unity. Thus, in a linear bias model, the bias factor for these galaxies would be $b_{opt} \simeq 1/\sigma_8$; for other galaxy populations, $b_{gal}\sigma_8$ may differ from unity.

Figure 1 shows the quantity $(d\sigma^2/d\ln k)^{1/2}/\sigma_8 = k^{3/2}\langle|\delta_k(t_0)|^2\rangle^{1/2}/\sqrt{2}\pi\sigma_8$, the contribution of the logarithmic interval around wavenumber k to the rms density fluctuation. The curves are labelled by the index n_s = 1, 0.6, 0.2, ..., -1. For fixed σ_8, as n_s is decreased from 1, the large scale power grows and the small scale power falls. At small $k \ll k_{eq}$, where $T(k) = 1$, the curves go as $k^{(n_s+3)/2}$. This figure also indicates the range of wavenumber probed by different observations of large-scale structure such as the CMBR anisotropy (the large-angle measurements of COBE and the $\sim 1°$ experiments at the South Pole), the galaxy angular correlation function $w_{gg}(\theta)$ inferred from deep photometric surveys, the power spectrum of IRAS galaxies from the QDOT and 1.2 Jy redshift surveys, and the large-scale streaming velocities (LSSV) reconstructed from galaxy redshift-distance surveys. On large scales, the dominant contribution to the CMBR anisotropy is the Sachs-Wolfe effect, the redshift suffered by CMBR photons climbing out of gravitational potential wells at the time of last scattering, $\Delta T/T \simeq (1/3)\Delta\phi$. Thus, through Poisson's equation, COBE essentially measures the density field on large scales, and directly probes the *primordial* power spectrum (the region where $T(k) \simeq 1$). On the other hand, the smaller scale observations use galaxies to probe the mass and are made in the region where the transfer function is non-trivial; caution must be used when inferring a density spectrum from the galaxy spectrum because of the possible complexities of biasing.

An example of these observational tests is shown in Fig. 2, which compares the galaxy angular correlation function $w_{gg}(\theta)$ determined from the APM galaxy survey (dots) with the models, again labelled by the spectral index n_s = 1, 0.8, 0.6, 0.4, ..., -1. While the vertical spread in the dots gives a rough measure of the statistical errors in the APM data, the vertical hatchmarks indicate the allowed region once corrections for possible systematic errors are included. The APM data has been scaled back to the

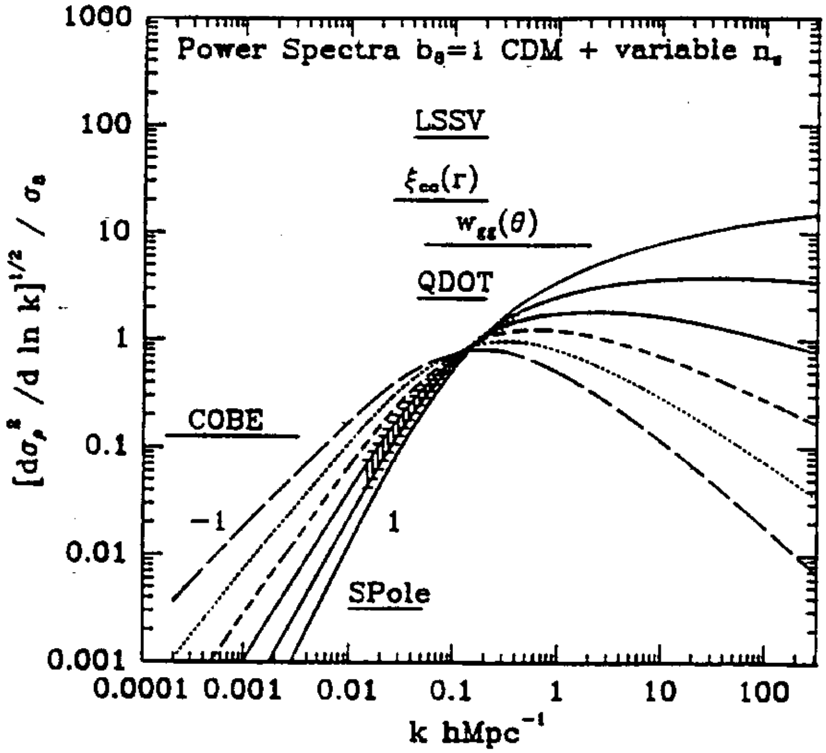

Figure 1. Perturbation power spectra for CDM models with variable index n_s, all normalized to $\sigma_8 = 1$.

depth of the Lick catalog, so that 1° corresponds roughly to a physical scale of $\sim 5h^{-1}$ Mpc. For the models, we use the fact that $w(\theta)$ is the two-dimensional projection of the spatial two-point correlation function, $\xi(r)$, which in turn is the Fourier transform of the galaxy power spectrum $P_{gal}(k)$. Fig. 2 indicates that a spectral index in the range $0 \lesssim n_s \lesssim 0.6$ is needed for the CDM model to fit the observations. (This range is also indicated by the vertical hatchmarks in Fig. 1.) In comparing the models with the data, we have assumed linear biasing, $b_g\sigma_8 = 1$, so this result is independent of the amplitude σ_8. As mentioned above, a similar fit to the data would be obtained by keeping $n_s = 1$ and modifying the transfer function by taking $\Omega_0 h = 0.2$, or by taking the 70/30 split of hot and cold dark matter, or by introducing a suitably scale-dependent bias.

Thus, observations of the large-scale galaxy distribution indicate that models with more relative large-scale power, such as CDM with $n_s \lesssim 0.6$, are preferred over SI CDM. However, as Fig. 1 shows, large-angle CMBR measurements such as COBE provide the strongest lever arm to test models with extra large-scale power. This is especially true for the $n_s < 1$ models (and less so for the other extra power fixes), where the power relative to the SI spectrum keeps growing at large scales. The COBE DMR team published (Smoot, etal. 1992) three pieces of information which can be used to constrain the density power spectrum and amplitude: the quadrupole anisotropy, $(\Delta T/T)_{l=2} \simeq 4.75 \times 10^{-6}(1 \pm 0.31)$, the rms temperature fluctuation on 10°, $\sigma_T(10°) = 1.085 \times 10^{-5}(1 \pm 0.169)$, and the temperature angular correlation function $C(\theta)$.

Figure 3 (solid curves) shows the constraint from $\sigma_T(10°)$ on the perturbation amplitude σ_8 as a function of the spectral index n_s for CDM. The error bars are combinations of the 1 sigma measurement error and the theoretical dispersion or 'cosmic variance'. Since the combined error on the quadrupole is larger, the corresponding constraint is

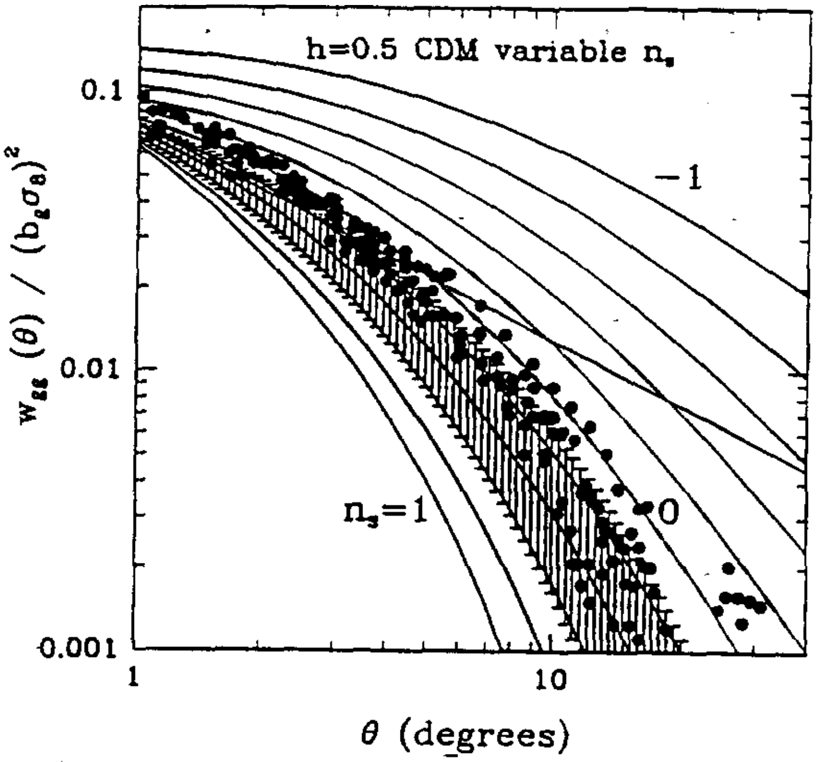

Figure 2. Angular correlation function for CDM models with variable $n_s = 1$, 0.8, 0.6, ..., -1, compared with APM survey data (dots and vertical hatch-marks).

less restrictive; on the other hand, χ^2 fits to the DMR correlation function $C(\theta)$ yield constraints on $\sigma_8(n_s)$ nearly identical to those in Fig. 3. A good analytic fit to the constraint in Fig. 3 is given by $\sigma_8 \simeq 1.17e^{-2.63(1-n_s)}$ $[1 \pm 0.2]$. In particular, for the spectral range favored by the APM data, $n_s \lesssim 0.6$, the DMR result requires $\sigma_8 \lesssim 0.5$. Note that the above estimate for σ_8 should be revised downward by $\sim$20% when the effects of baryons and the radiation density are properly included. Thus, for the SI spectrum ($n_s = 1$), the COBE result is consistent with an unbiased model in the sense that $\sigma_8 \simeq 1$, while progressively larger deviations from SI require larger bias.

The fact that COBE requires a low σ_8 amplitude for $n_s < 1$ models is helpful in one sense: it reduces the pair-wise velocities of galaxies on small scales from the unbiased SI CDM value of $\sim$ 1000 km/sec closer to the observed value of $\sim$ 300 km/sec (Gelb, Gradwohl, and Frieman 1993). On the other hand, if the amplitude is made too small, it becomes problematic for galaxy formation: for small σ_8, galaxy halos form at very low redshift, making it difficult to understand high-redshift galaxies. In fact, requiring that galaxies form at redshift $z_{gf} > 2$ and using the COBE $\sigma_T(10^\circ)$ normalization of $\sigma_8(n_s)$ yields the constraint $n_s \gtrsim 0.6$, which is at least marginally beyond the region which fits the APM data. In addition, reproducing the observed large-scale velocities inferred e.g., from POTENT is problematic in this case. These results suggest that a non-SI spectrum by itself does not remedy all the difficulties of CDM, but that a moderate deviation from scale-invariance can ameloriate them to some degree.

The COBE results above apply for models like natural inflation. For extended and power law inflation, which also give rise to power-law perturbation spectra, the situation

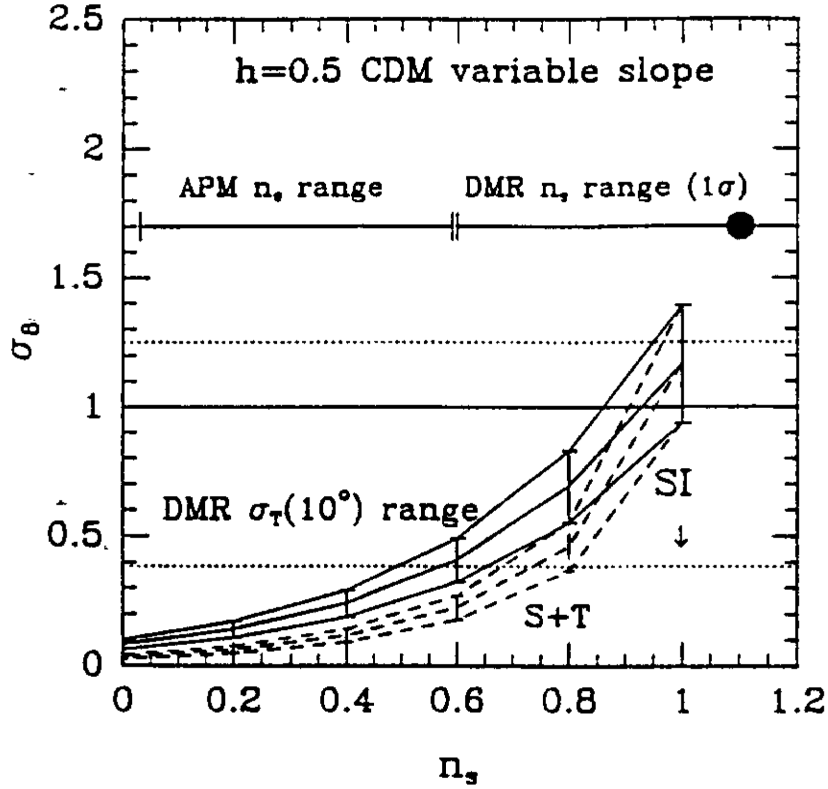

Figure 3. Constraint on the amplitude σ_8 for CDM models as a function of n_s from COBE's rms fluctuations on 10°. Also indicated is the APM allowed range $0 \leq n_s \leq 0.6$ and the 1σ range suggested by the DMR team from their fit to $C(\theta)$, $n_s \simeq 1.1 \pm 0.5$.

is somewhat different: in this case, gravity waves contribute a significant fraction of the COBE signal when $n_s < 1$ (Davis etal 1992), implying lower values for σ_8 (dashed lines marked S+T in Fig. 3); e.g., for $n_s < 0.6$, we now have $\sigma_8 < 0.27$. For values of n_s significantly below 1, this exacerbates the problem of early galaxy formation; combining $z_{gf} > 2$ with COBE's $\sigma_T(10^o)$ now yields the bound $n_s > 0.76$, well outside the range $n_s < 0.6$ needed for consistency with APM. On the other hand, the lower σ_8 amplitudes implied for these models with even a small deviation from SI (say, $n_s \sim 0.8$) would be useful in bringing down small-scale velocities. The flatness of the large-scale spectrum implied by the APM and IRAS results would, however, have to be achieved by other means. The combination of data from COBE with results now starting to come from anisotropy measurements on smaller angular scales (Gaier, etal. 1992) may allow the disentangling of the scalar and tensor contributions to the CMBR anisotropy, and this would be a powerful test of inflationary models (Copeland etal. 1993, Crittenden etal. 1993, Turner 1993). Finally, it should be noted that the COBE DMR temperature map appears to correlate well with the results of the MIT balloon experiment (Meyer 1992), which covers a smaller area of the sky with better angular resolution–providing confirmation that the DMR signal is intrinsic to the sky. When subsequent years of the DMR data are analyzed, the kind of analysis above will be further refined, allowing us to further zero in on the viable models for large-scale structure.

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